International Civil Aviation Organization

The Twenty-First Meeting of the Regional Airspace Safety Monitoring Advisory Group (RASMAG/21)

Bangkok, Thailand, 14-17 June 2016

Agenda Item 3:   Reports from Asia/Pacific RMAs and EMAs

BOBASMA HORIZONTAL SAFETY MONITORING REPORT

(Presented by BOBASMA/India)

SUMMARY

This paper presents the periodic safety assessment for the implementation of 50 NM & 30 NM Reduced Horizontal Separation in the Bay of Bengal Arabian Sea Indian Ocean airspace for the period January to December 2015.

1.   INTRODUCTION

1.1   The annual safety review for the continued safe use of 50 NM reduced horizontal separation (RHS) on sixteen routes and 30 NM reduced horizontal separation on four routes in the Bay of Bengal Arabian Sea Indian Ocean airspace was conducted using the TSD of December 2015 of Chennai, Mumbai, Kolkata, Colombo, Kuala Lumpur and Yangon FIRs. The Large Lateral Deviation (LLD) and Large Longitudinal Error (LLE) reports received by BOBASMA for the period 1st January 2014 to 31st December 2014 was also used for conducting the safety assessment.

2.   DISCUSSION

2.1   The report of the safety assessment conducted for monitoring the horizontal safety risks is attached as Appendix – A.

Executive Summary

2.2 **Table 1** provides the Bay of Bengal Arabian Sea Indian Ocean airspace horizontal risk estimates. **Figure 1** presents the lateral and longitudinal collision risk estimate trends for BOBASIO airspace during the period January 2015 to December 2015.

Bay of Bengal Arabian Sea Indian Ocean Airspace– estimated annual flying hours = 556,354 hours *(note: estimated hours based on Dec 2015 traffic sample data)*			
Risk	**Risk Estimation**	**TLS**	**Remarks**
RASMAG 20 Lateral Risk	*1.07856×10^{-9}*	*5.0×10^{-9}*	*Below TLS*
RASMAG 20 50 NM Longitudinal Risk	*1.59734×10^{-9}*	*5.0×10^{-9}*	*Below TLS*
RASMAG 20 30 NM Longitudinal Risk	*0.127551×10^{-9}*	*5.0×10^{-9}*	*Below TLS*
Lateral Risk	**1.69506×10^{-9}**	5.0×10^{-9}	**Below TLS**
50 NM Longitudinal Risk	**3.97378×10^{-9}**	5.0×10^{-9}	**Below TLS**
30 NM Longitudinal Risk	**0.138395×10^{-9}**	5.0×10^{-9}	**Below TLS**

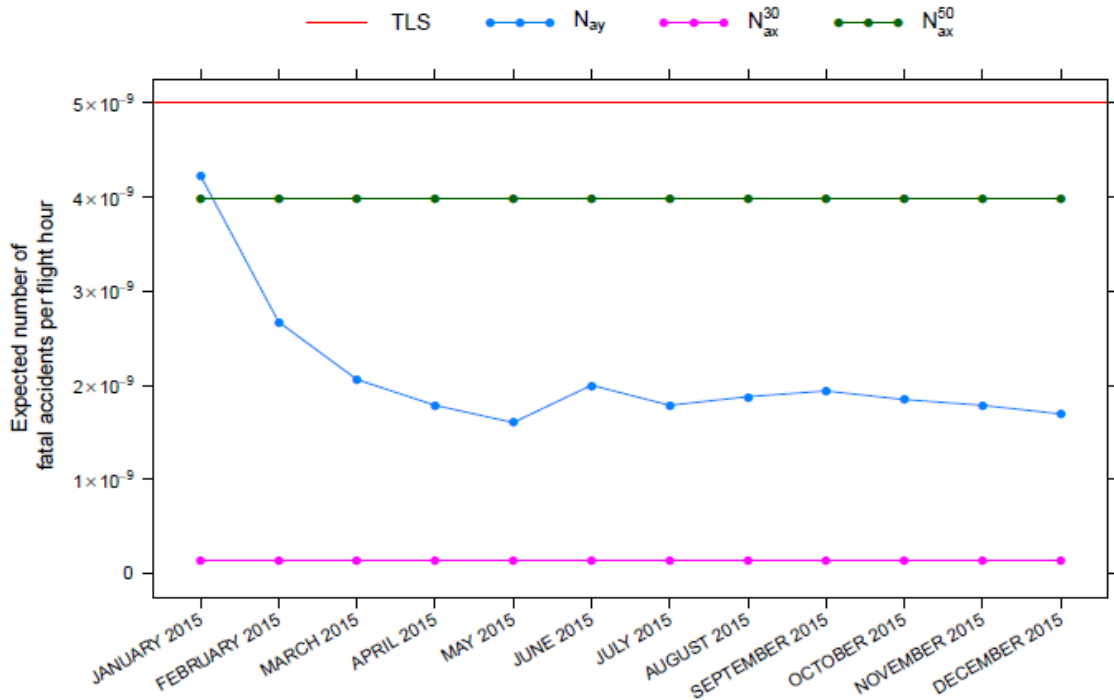**Table 1:** Bay of Bengal Arabian Sea Indian Ocean Airspace Horizontal Risk Estimates

Figure 11: Assessment of Compliance with Lateral and Longitudinal TLS Values.

Figure 1: Bay of Bengal Arabian Sea Indian Ocean Airspace Horizontal Risk Estimates

2.3   **Table 2** contains a summary of Large Lateral Deviations (LLD) and Large Longitudinal Errors (LLE) received by BOBASMA for the BOBASIO airspace.

Code	Deviation Description	No.
A	Flight crew deviates without ATC Clearance	0
B	Flight crew incorrect operation or interpretation of airborne equipment	0
D	ATC system loop error	0
E	Coordination errors in the ATC unit-to-ATC unit transfer of control responsibility	6
G	Turbulence or other weather related causes	2
Total		8

Table 2: Summary of BOBASIO XX Airspace LLD and LLE Reports

2.4   The six Category E LLDs occurred when aircraft on entering an FIR reported abeam the entry point and there was no information from the transferring unit of such deviations. Two Category G deviation occurred when aircraft deviated due bad weather.

3.   ACTION BY THE MEETING

3.1 The meeting is invited to:

a)   note the information contained in this paper; and

b)   discuss any relevant matters as appropriate.

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APPENDIX – A

Lateral and Longitudinal Collision Risk
Assessment of Bay of Bengal Arabian Sea Indian Ocean Airspace

1.   Introduction

In this article we reinvestigate the collision risk between two aircraft flying over the Bay of Bengal, Arabian Sea, and Indian Ocean (BOBASIO) region.   This safety assessment is undertaken jointly by the Airports Authority of India (AAI) and the Indian Statistical Institute, Delhi Centre under the MoA signed between the two organizations on May 16, 2015. The goal of this study is to confirm that the Target Level of Safety (TLS) which is $5 \times 10^{-9 \, fatal}$ accidents per flight hour, is currently met.

We first observe that in the BOBASIO region, different separation standards are currently present. In particular, the following routes have different longitudinal separation standards:

- 30 NM: On the routes **M300, N571, P570 and P574** (started from September 18, 2014);

- 50 NM: On the routes **L301, L507, L510, L759, M770, N563, N877, N895, P628, P646 and P762**.

For this reason, two separate longitudinal analyses for these two sets of routes. However, the lateral separation between two parallel routes is at least 50NM for all routes. So we carried out a combined lateral risk analysis.

In this article we carry out the quantitative risk analysis based on two types of datasets supplied by six FIRs in the region.

- **Traffic Sample Data (TSD)**:

  Traffic sample data from Chennai, Kolkata, Mumbai, Colombo, Kuala Lumpur and Yangon FIRs for the month of December 2015 was used. The original data contained several anomalies, which we tried to detect and remove. Briefly, the following initial filtering criteria were used:

  – Duplicate records were removed.

  – Records with Exit time less than Entry time were removed.

  – Records with flight level less than F280 were removed.

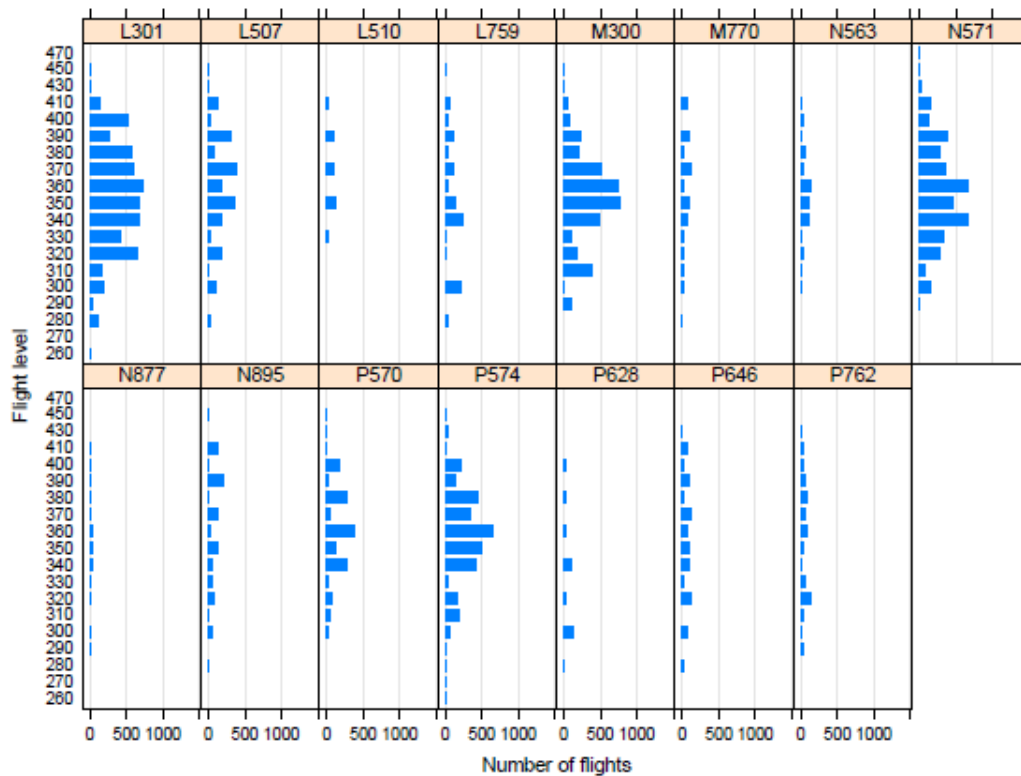 – Records with unusually high or low traversal times were removed.

Figure 1: Number of flights by route and flight level based on December 2015 TSD.

40794 records that were retained after filtering were considered for the subsequent statistical analysis. Figure 1 provides a graphical summary of the number of flights by route and flight level for RHS routes.

- **Gross Navigational Error (GNE) Data:**

Reports of Gross Navigational Errors for the preceding twelve month period were received from Chennai, Kolkata, and Mumbai, as summarized in Table 1.

Year	Month	FIR	Flights	LLD	LLE
2015	JANUARY	CHENNAI	5282	0	0
2015	FEBRUARY	CHENNAI	4863	0	0
2015	MARCH	CHENNAI	5215	0	0
2015	APRIL	CHENNAI	5234	0	0
2015	MAY	CHENNAI	5265	0	0
2015	JUNE	CHENNAI	5405	1	0
2015	JULY	CHENNAI	5379	0	0
2015	AUGUST	CHENNAI	5454	0	0
2015	SEPTEMBER	CHENNAI	5067	0	0
2015	OCTOBER	CHENNAI	5246	0	0
2015	NOVEMBER	CHENNAI	4658	0	0
2015	DECEMBER	CHENNAI	5459	0	0
2015	JANUARY	KOLKATA	2542	0	0

2015	FEBRUARY	KOLKATA	2253	0	0
2015	MARCH	KOLKATA	2450	0	0
2015	APRIL	KOLKATA	2179	0	0
2015	MAY	KOLKATA	2134	0	0
2015	JUNE	KOLKATA	1996	0	0
2015	JULY	KOLKATA	2292	0	0
2015	AUGUST	KOLKATA	2196	1	0
2015	SEPTEMBER	KOLKATA	2106	2	0
2015	OCTOBER	KOLKATA	2191	0	0
2015	NOVEMBER	KOLKATA	2283	0	0
2015	DECEMBER	KOLKATA	2562	0	0
2015	JANUARY	MUMBAI	13903	1	0
2015	FEBRUARY	MUMBAI	12004	0	0
2015	MARCH	MUMBAI	14029	0	0
2015	APRIL	MUMBAI	11613	0	0
2015	MAY	MUMBAI	14248	0	0
2015	JUNE	MUMBAI	17874	2	0
2015	JULY	MUMBAI	20498	0	0
2015	AUGUST	MUMBAI	20350	1	0
2015	SEPTEMBER	MUMBAI	20002	0	0
2015	OCTOBER	MUMBAI	13154	0	0
2015	NOVEMBER	MUMBAI	12884	0	0
2015	DECEMBER	MUMBAI	13042	0	0
Total			273312	8	0

Table 1: Summary of reports of Gross Navigational Errors.

In Section 2 we discuss the risk assessment for the lateral direction and Section 3 gives the same for the longitudinal direction.

2. Lateral Collision Risk Assessment

2.1 Lateral Collision Risk Model

In order to compute the *level of safety* for lateral deviations of operations in the BOBASIO region we use the *Reich Lateral Collision Risk Model*. It models the lateral collision risk due to the loss of lateral separation between aircraft on adjacent parallel tracks flying at the same flight level. The model is as follows:

$$N_{ay} = P_y(S_y) P_z(0) \frac{\lambda_x}{S_x} \left\{ E_y(\text{same}) \left[\frac{\overline{|\Delta V|}}{2\lambda_x} + \frac{\overline{|\dot{y}(S_y)|}}{2\lambda_y} + \frac{\overline{|\dot{z}|}}{2\lambda_z} \right] + E_y(\text{opp}) \left[\frac{2\overline{|V|}}{2\lambda_x} + \frac{\overline{|\dot{y}(S_y)|}}{2\lambda_y} + \frac{\overline{|\dot{z}|}}{2\lambda_z} \right] \right\}. \tag{1}$$

The parameters in the equation (1) are defined as follows:

- *Nay* := Expected number of fatal accidents (two for every collision) per flight hour due to the loss of lateral separation between co-altitude aircrafts flying on tracks with planned S_y NM lateral separation.

- *Sy:* = Minimum planned lateral separation.

6

- λx := Average length of an aircraft flying in BOBASIO region.

- λy := Average wingspan of an aircraft flying on BOBASIO region.

- λz := Average height of an aircraft flying on BOBASIO region.

- $P_y (S_y)$:= The probability of lateral overlap of aircraft nominally flying on adjacent flight paths, separated by S_y.

- $P_z (0)$:= Probability of vertical overlap of two aircrafts assigned the same flight path at the same flight level.

- Sx: = Length of half the interval in NM used to count proximate aircraft at adjacent routes.

- Ey (same):= Same direction lateral occupancy at same assigned flight level.

- Ey (opp):= Opposite direction lateral occupancy at same assigned flight level.

- $\overline{|\Delta V|}$: = Average relative speed of two aircraft flying on parallel routes in same direction.

- |V |:= Average ground speed on an aircraft.

- $\overline{|\dot{y}(S_y)|}$: = Average relative lateral speed of aircraft pair at loss of planned lateral separation of Sy.

- $\overline{|\dot{z}|}$: = Average relative vertical speed of a co-altitude aircraft pair assigned to the same route.

A collision, and consequently two fatal accidents, can only occur if there is overlap between two aircraft in all three dimensions simultaneously. Equation (1) gathers the product of the probabilities of losing separation in each one of the three dimensions.

As it has already been said, $P_z \overline{(0)}$ is the probability of vertical overlap; $P_y (S_y)$ is the probability of lateral overlap and the combinations of $\frac{\lambda_x}{S_x} E_y \text{ (same)}$ and $\frac{\lambda_x}{S_x} E_y \text{ (opp)}$ relate to the probability of longitudinal overlap of aircraft on adjacent parallel tracks and at the same flight level. All the probabilities can be interpreted as proportions of flight time in the airspace during which overlap in the pertinent dimension occurs. As the collision risk is expressed as the expected number of fatal accidents per flight hour, the joint overlap probability must be converted into number of events involving joint overlap in the three dimensions, relating overlap probability with *passing frequency*. Here we note that passing frequency between two adjacent routes is the average number of events, per flight hour, in which two aircraft are in longitudinal overlap when travelling in the opposite or same direction at the same flight level. This is achieved by means of the expressions within square brackets in Equation (1). Each of the terms within square brackets represents the reciprocal of the average duration of an overlap in one of the dimensions. For example, $\frac{\overline{|\Delta V|}}{2\lambda_x}$ is the reciprocal of the average duration of an overlap in the longitudinal direction for same direction traffic. In the case of longitudinal direction too, but for opposite direction, the average relative speed is $2\overline{V}$ and the average overlap time $\frac{2\overline{|V|}}{2\lambda_x}$.

The model is based on the following hypothesis:

- All routes are parallel. [1]

- All collisions normally occur between aircraft on adjacent routes, although, if the probability of overlap is significantly large, they may also occur on non-adjacent routes.

- The entry times into the track system are statistically independent.

- The lateral deviations of aircraft on adjacent tracks are statistically independent.

- The vertical, longitudinal and lateral deviations of an aircraft are statistically independent.

- The aircraft are replaced by rectangular boxes.

- There is no corrective action by pilots or ATC when two aircraft are about to collide.

The model also assumes that the nature of the events making up the lateral collision risk is completely random. This implies that any location within the system can be used to collect a representative data sample on the performance of the system.

2.2 Estimated Values of the Parameters and Estimated Lateral Collision Risk

The following table gives the values of the parameters of the right-hand side of the equation (1) which are obtained from our analysis.

(*Note: In the BOBASIO region there are cross routes, such as, P762. A stricter time separation is imposed on such a route but we ignore that to be more conservative about our estimates*).

Parameter	Estimated Values	Source of the Estimate
S_y	50 NM	Current minimum lateral separation.
λ_x	0.03192448 NM	Estimated from TSD (see Section 2.3).
λ_y	0.02966417 NM	Estimated from TSD (see Section 2.3).
λ_z	0.008968392 NM	Estimated from TSD (see Section 2.3).
$P_y(50)$	4.16819×10^{-8}	Estimated using a mixture model (see Section 2.4).
$P_z(0)$	0.538	Conservative value used in previous safety assessments (see Section 2.5).
S_x	50 NM	Conservative value, taken to be the current longitudinal separation in all but four routes.
E_y (same)	0.06569784	Estimated from the TSD (see Section 2.6).
E_y (opp)	0	No opposite directional lateral occupancy at same assigned flight level.
$|\Delta V|$	29 knots	Value obtained from TSD (see Section 2.7).
$|\dot{y}(50)|$	75 knots	Conservative value taken from EMA Handbook (see Section 2.8).
$|\dot{z}|$	1.5 knots	Conservative value as per EMA Handbook (see Section 2.9).

Finally this leads to the following estimate for the lateral collision risk N_{ay}.

$$N_{ay} = 1.69506 \times 10^{-9}$$

2.3 Estimating Average Aircraft Dimensions

Table 2 summarizes the distribution of aircraft population in the TSD. To be conservative, we used the maximum aircraft dimensions.

Type	Length	Wingspan	Height	Flights
B77W	73.9	64.8	18.5	7819
B738	39.2	34.4	12.57	4876

A333	63.6	60.3	16.85	4607
A320	37.57	34.1	11.76	3581
A332	58.8	60.3	17.4	2817
A388	73	79.8	24.1	2853
B772	63.7	60.9	18.4	1786
A321	44.51	34.1	11.76	1498
B744	70.6	64.8	19.4	1082
A343	63.6	60.3	16.85	700
B763	54.9	47.6	15.9	662
B77L	63.7	64.8	18.3	599
B773	73.9	60.9	18.4	451
A346	75.3	63.45	17.3	402
A319	33.84	34.1	11.76	303
MD11	61.2	51.7	17.6	125
A310	46.66	43.9	15.8	67
B752	47.3	38.1	13.6	42
GLEX	30.3	26.9	7.6	40
CL60	20.85	19.6	6.3	33
GLF4	26.9	23.7	7.4	32
B737	33.6	34.3	12.6	24
GLF5	29.4	28.5	7.5	22
GL5T	28.69	28.65	7.7	10
B74S	56.3	59.6	20	22
F2TH	20.2	19.3	7.1	14
H25B	15.6	15.7	5.4	12
F900	20.2	19.3	7.6	12
A380	73	79.8	24.1	11
GL5T	28.69	28.65	7.70	10
A306	54.10	44.84	16.54	10
C17	53.00	51.80	16.80	5
B742	70.60	59.60	19.30	5

Table 2: Dimensions of aircraft types, along with number of records in the TSD

2.4   Estimating Probability of Lateral Overlap: $P_y(S_y)$

The probability of lateral overlap of aircraft nominally flying on adjacent flight paths, separated by S_y, is denoted by $P_y(S_y)$ and is defined as

$$P_y(S_y) := \mathbf{P}\left(|S_y + Y_1 - Y_2| \le \lambda_y\right), \qquad (2)$$

where Y_1 and Y_2 are assumed to be the lateral deviations of two aircraft which are nominally separated by S_y. We assume that Y_1 and Y_2 are identically distributed but statistically independent with a distribution F_y.

We model F_y as mixture distribution having a *core* distribution G_y and a *non-core* distribution H_y.

- The core distribution G_y, represents errors that derive from standard navigation system deviations. These errors are always present, as navigation systems are not perfect and they have a certain precision.

- The non-core distribution H_y, represents *Gross Navigation Errors (GNE)*, that corresponds to what may be viewed as non-nominal performance.

We assume that a standard navigation system error represented by the core distribution may take large values but the non-core distribution representing gross navigation errors can only take large values. But in most cases it is impossible to determine with certainty if a given observed lateral error arose from the core or from the tail term of the distribution. Therefore, the overall lateral deviation distribution is modeled as:

$$F_y(y) = (1 - \alpha)\,G_y(y) + \alpha H_y(y) \quad\ldots\ldots (3)$$

The mixing parameter α is the probability of a *gross navigational error*.

The core lateral deviation distribution G_y is modeled by a *Double Exponential* distribution with a parameter $\beta_y > 0$ as the rate, that is, if $Y_1 \sim G_y$ then

$$\mathbf{P}(|Y_1| > y) = e^{-\beta_y\,y},$$

where $y \ge 0$. In other words we assume that the core distribution has a density of the form

$$g_y(y) = \frac{\beta_y}{2} e^{-\beta_y |y|}.$$

Finally the non-core distribution H_y is modeled by a "*Separated Double Exponential*" distribution with parameters $\mu_y > 0$, representing the "separation and $\gamma_y > 0$ the rate parameter, that is, if $Y_2 \sim H_y$ then

$$\mathbf{P}(Y_2 > \mu_y + y) = \frac{1}{2} e^{-\gamma_y y} \quad \text{and}$$

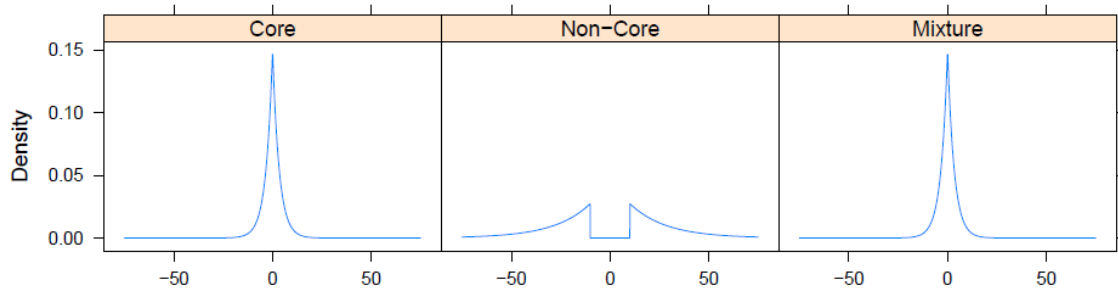$$\mathbf{P}(Y_2 < -\mu_y - y) = \frac{1}{2} e^{\gamma_y y},$$

Figure 2: Modeling of lateral deviation.

where $y \geq 0$. This really means that the non-core distribution H_y gives no mass in $[-\mu_y, \mu_y]$ and outside it decays as a Double Exponential distribution with rate parameter γ_y. The density of this distribution is given by

$$h_y(y) = \begin{cases} \frac{\gamma_y}{2} e^{\gamma_y(y+\mu_y)} & \text{if} \quad y < -\mu_y \\ 0 & \text{if} \quad -\mu_y \leq y \leq \mu_y \\ \frac{\gamma_y}{2} e^{-\gamma_y(y-\mu_y)} & \text{if} \quad y > \mu_y \end{cases} .$$

This modeling is similar to what has been used by FAA and also in EUR/SAM except here we take a double exponential distribution, namely the core distribution to explain all the typical and atypical errors which are not a gross navigational error, and use the separated double exponential distribution for the gross navigational errors. This in turn gives a better understanding of the mixing parameter α which we estimate by taking the 95% *upper confidence limit* from the available GNE data. This confidence limit does not have a nice formula when one or more GNEs are observed, but can be computed using numerical methods. The value comes out to be

$$\hat{\alpha} = \textbf{5.2 x 10}^{\textbf{-5}}$$

Here we would like to note that even though the non-core distribution Hy has a discontinuous density hy, it does not create difficulty in this risk assessment.

The parameter βy is estimated under the RNP10 assumption of ±10 NM deviation with 95% confidence, this leads to the estimate

$$\widehat{\beta}_y = \frac{-\log 0.05}{10} = 0.2995732$$

The parameter μy is taken to be 10 based on RNP10 consideration and γy is then estimated by maximizing
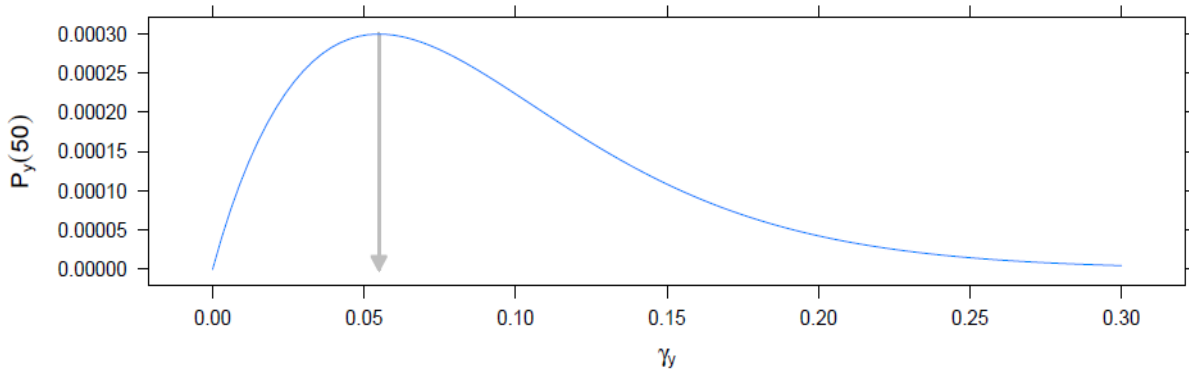
11

Figure 3: Wingspan overlap probability as a function of γy with $Sy = 50$ NM initial separation.

the wingspan overlap probability with $Sy = 50$ NM initial separation (see Figure 3). This is a conservative method similar to what has been used by FAA and also in EUR/SAM. The estimated value of γy is 0.05489708 leading to the estimated value of $P_y(50)$ as 4.16819×10^{-8}.

To be conservative, we also considered the possibility of unreported GNEs, and computed the estimates of $P_y(50)$ and Nay had we observed more GNEs. The results, given below, are still well below the TLS.

Note that the actual number of GNEs observed was 8.

No. of GNEs	$Py(50)$	Nay
8	4.16819×10^{-8}	1.69506×10^{-9}
9	4.39313×10^{-8}	1.78653×10^{-9}
10	4.54309×10^{-8}	1.84751×10^{-9}
11	4.69305×10^{-8}	1.9085×10^{-9}
12	4.91799×10^{-8}	1.99997×10^{-9}

2.5   Estimating Probability of Vertical Overlap: $P_z(0)$

The probability of vertical overlap of aircraft nominally flying at the same flight level on laterally adjacent flight paths is denoted by $P_z(0)$. It is defined by

$$P_z(0) = \mathbf{P}(|Z1 - Z2| \leq \lambda_z),$$

where $Z1$ and $Z2$ are the height deviations of two aircraft nominally flying at the same flight levels on laterally adjacent flight paths.

We assume that $Z1$ and $Z2$ are statistically independent with distribution F_z. Unlike in the computation of $P_y(Sy)$ where we assume the lateral deviations follow a mixture distribution here we may assume that F_z is a Double Exponential distribution with parameter $\beta_z > 0$, that is, with density function

$$f_z(z) = \frac{\beta_z}{2} e^{-\beta_z |z|}$$

One can then estimate $\beta_z > 0$ by

$$\widehat{\beta_z} = -\frac{\log 0.05}{0.032915} = 91.0142.$$

This is under assumption that a typical aircraft stays within ±200 ft = ±0.032915 NM of its assigned flight level 95% of the time. This leads to an estimated value 0.3523139 for $P_z(0)$.

Unfortunately this analysis ignores both the effect of large height deviations (LHDs) and aircraft altimetry system errors (ASE) which are not estimable directly. So we use a conservative value of 0.538, as used by MAAR for vertical safety assessment in BOB region.

2.6   Estimating the Lateral Occupancy Parameters: E_y (same) and E_y (opp)

In equation (1) there are two occupancy terms, one for same direction occupancy E_y (same) and another one for opposite direction occupancy E_y (opp).

Same direction occupancy is defined as the average number of aircraft that are, in relation to a typical aircraft

- flying in the same direction as it;

- nominally flying on tracks one lateral separation standard away;

- nominally at the same flight level as it; and

- within a longitudinal segment centered on it.

The length of the longitudinal segment, $2S_x$, is usually considered to be the length equivalent to 20 minutes of flight resulting to a value of 160 NM. It has been verified that the relationship between S_x and the occupancy is quite linear.

A similar set of criteria can be used to define opposite direction occupancy, just replacing "flying in the same direction" by "flying in the opposite direction". Occupancy, in general, relates to the longitudinal

WP1	WP2	Proximate	Total
BIDEX	ORARA	6	882
IGOGU	IGREX	108	2840
NOPEK	IGOGU	106	3204
GIRNA	IDASO	110	3270
VATLA	ORARA	6	762
IGOGU	EMRAN	0	2266
LIBDI	MEPEL	0	648
RINDA	SAGOD	26	4034
MEPEL	IBITA	10	1462
IBITA	TEBOV	236	4232
SAGOD	IBITA	8	1542
POMAN	IGAMA	758	6274
KITAL	LOTAV	208	4730
OPIRA	IGAMA	0	3826

LOTAV	REXOD	126	4572
TOTOX	REXOD	134	2766
TOTOX	PARAR	630	5384
ADPOP	SUGID	0	4168
RASKI	PARAR	1200	7522
NOBAT	SUGID	1864	10282
POMAN	ODOLI	0	2448
KITAL	ASPUX	0	484
NISOK	NIXUL	0	410
NIXUL	TOPIN	52	586
SULTO	DUGOS	0	782
ATETA	DEMON	0	174
UDULO	KAKIB	0	646
BUBKO	DOPID	70	2614
VATLA	MABUR	0	496
SAGOD	MEPEL	2	572
DWI	BETNO	0	790
MIPAK	LALAT	0	1484

Table 3: Number of laterally proximate flights per route pair, based on TSD.

overlap probability and can be obtained by the equation

$$E_{y=} \frac{2T_y}{H}$$

where T_y represents the total proximity time generated in the system and H is the total flight hours generated in the system during the considered period of time.

We estimate this quantity by direct estimation from time at waypoint passing using the TSD. For this we compute the number of proximate pairs by comparing the time at which an aircraft on one route passes a waypoint with the time at which another aircraft on a parallel route passes the homologous waypoint. When
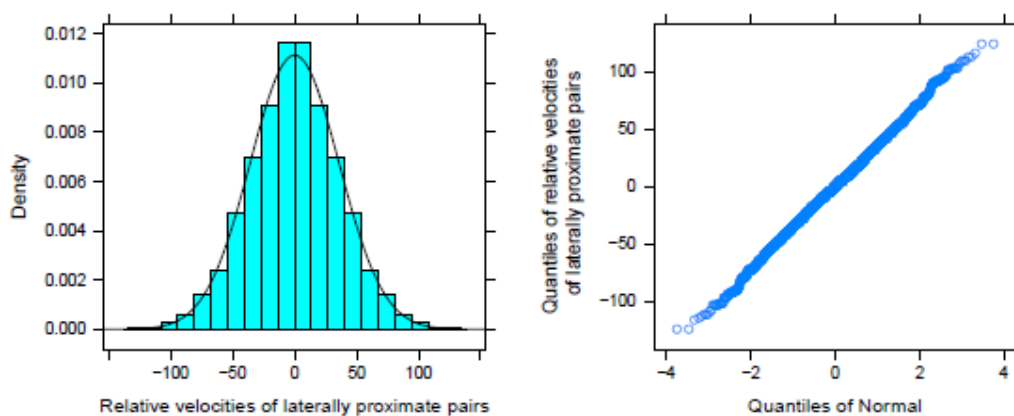

Figure 4: Distribution of relative velocities of laterally proximate pairs. The Normal distribution with sample standard deviation looks like a reasonable fit.

14

the difference between passing times is less than certain value, 10 minutes in this case, it is considered that there is a proximate pair in that pair of routes. Occupancy is then calculated using the following expression:

$$E_{Y=} \frac{2n_y}{n}$$

where the numerator n_y is the number of proximate pairs and the denominator, n, is the the total number of aircraft. The observed number of proximate pairs and the total number of flights per route pair are summarized in Table 3.

2.7   Estimate of Average Relative Longitudinal Speed: $\overline{|\Delta V|}$

$\overline{|\Delta V|}$ is the average relative longitudinal speed between aircraft flying in the same direction. We estimate it from the TSD by taking the differences between the speeds of all the pairs of aircraft that constitute a lateral proximate pair in the same direction (see Figure 4). $\overline{|\Delta V|}$ is estimated as the mean absolute value of all the calculated differences, which turns out to be 28.47588. We use the conservative value 29. Here we note that the lateral proximate pairs are already determined while estimating the parameter E_y (same).

2.8   Estimate of Average Relative Lateral Speed: $\overline{|\dot{y}(S_y)\|}$

$\overline{|\dot{y}(S_y)\|}$ is the average relative lateral cross-track speed between aircraft, flying on adjacent routes separated by S_y NM at the same flight level, that have lost their lateral separation. The estimation of this parameter generally involves the extrapolation of radar data, speeds and lateral deviations, but such radar data were not available for this study. So we take a conservative value 75 knots as per the EMA Handbook.

2.9   Estimate of Average Relative Vertical Speed : $\overline{|\dot{z}|}$

$\overline{|\dot{z}|}$ denotes the average modulus of the relative vertical speed between a pair of aircraft on the same flight level of adjacent tracks that has lost lateral separation. It is generally assumed that $\overline{|\dot{z}|}$ is independent of the size of the lateral separation between the aircraft and, for aircraft in level flight, it can also be considered that there is no dependency of $\overline{|\dot{z}|}$ with the vertical separation between the aircraft. As noted by various agencies data on $/\dot{z}/$ are relatively scarce but typically taken as 1.5 knots which is considered to be conservative (see EMA Handbook).

3   Longitudinal Collision Risk Assessment

In order to compute the *level of safety* for longitudinal deviations of operations on the BOBASIO region we use the *Longitudinal Collision Risk Model*. It models the longitudinal collision risk due to the loss of longitudinal separation between aircrafts flying on the same route at the same flight level. The model is as follows:

$$N_{ax} = P_y(0)\, P_z(0) \frac{2\lambda_x}{|\dot{x}|} \left(\frac{\overline{|\dot{x}|}}{2\lambda_x} + \frac{\overline{|\dot{y}(0)|}}{2\lambda_y} + \frac{\overline{|\dot{z}|}}{2\lambda_z} \right) \times \left[\sum_{k=m}^{M} 2Q(k)\, \mathbf{P}(K > k) \right]. \quad (4)$$

The parameters in the equation (4) as follows

- N_{ax} := Expected number of fatal accidents (two for every collision) per flight hour due to the loss of longitudinal separation between co-altitude aircrafts flying on the same track with planned minimum in NM longitudinal separation.

- m := Minimum longitudinal separation in NM.

- M := Maximum initial longitudinal separation between aircraft pair which will be monitored by ATC in order to prevent loss of longitudinal separation standard.

- λ_x := Average length of an aircraft flying on BOBASIO region.

- λ_y := Average wingspan of an aircraft flying on BOBASIO region.

- λ_z := Average height of an aircraft flying on BOBASIO region.

- $P_y(0)$:= Probability that two aircraft assigned at the same route will be at same across-track position.

- $P_z(0)$:= Probability that two aircraft assigned to same flight level are at same geometric height.

- $\overline{|\dot{x}|}$:= Minimum relative along-track speed necessary for following aircraft in a pair separated by in NM at a reporting point to overtake lead aircraft at the next reporting point.

- $\overline{|\dot{y}(0)|}$ = Relative across-track speed of same route aircraft pair.

- $\overline{|\dot{z}|}$ = Average relative vertical speed of a co-altitude aircraft pair assigned to the same route.

- Q(k):= Proportion of aircrafts for which the following aircraft has initial longitudinal separation k.

- $\mathbf{P}(K > k)$:= Probability that a pair of same route co-altitude aircraft with initial longitudinal separation k will lose at least as much as k longitudinal separation before correction by ATC.

Once again, a collision, and consequently two fatal accidents, can only occur if there is overlap between two aircraft in all three dimensions simultaneously. Equation (4) gathers the product of the probabilities of losing separation in each one of the three dimensions.

The equation is derived under similar assumption as done in the case of lateral collision risk assessment.

We should note here the the first part of the right-hand side of the equation (4) gives the probability of a collision given an event of overtake of a front aircraft by a behind aircraft when both are nominally flying at the same route at the same flight level, and the second part which is inside the square bracket is the expected number of aircrafts involved in such overtake events.

As different longitudinal separation standards are present, we carry out two different analyses: One is for the four routes, namely **M300, N571, P570**, and **P574**, presented in Section 3.1. The other is for the other **RNP10** routes in the region, namely **L301, L507, L510, L759, M770, N563, N877, N895, P628, P646**, and **P762**, presented in Section 3.2

3.1 Estimated Values of the Parameters and Estimated Longitudinal Collision Risk

The following table gives the values of the parameters of the right-hand side of the equation (4) which are obtained from the analysis

Parameter	Estimated Values	Source of the Estimate
m	30 NM	Current minimum longitudinal separation (due to RHS).
M	160 NM	Conservative value corresponding to 20 minutes separation.
λx	0.02962328 NM	Estimated using flights on 30NM routes only.
λy	0.02717288 NM	Estimated using flights on 30NM routes only.
λz	0.008430762 NM	Estimated using flights on 30NM routes only.
$P_y(0)$	0.2	Conservative estimate (see Section 3.1.1).
$P_z(0)$	0.538	Conservative value used in previous safety assessments (see Section 2.5).
$\lvert \dot{x} \rvert$	12 knots	Conservative estimate using speed and distance between way points (see Section 3.2.1).
$\lvert \dot{y}(0) \rvert$	1 knot	RASMAG/9 safety assessment (see Section
$\lvert \dot{z} \rvert$	1.5	Conservative value as per EMA Handbook (see Section 2.9).
$Q(k)$	See Table 4	Obtained from TSD (see Section 3.2.2).
$\mathbf{P}(K > k)$	See Table 4	Computed using normal model on speed (see Section 3.1.5).

Finally this leads to the following estimate for the longitudinal collision risk N_{ax}^{30}

$$N_{ax}^{30} = 0.138395 \times 10^{-9}$$

k (mins)	k (NM)	$Q(k)$	$P(K>k)$
1	8	0:00000000	1.91915×10^{-1}
2	16	0:00000000	7.47336×10^{-2}
3	24	0:00000000	2.50701×10^{-2}
4	32	0:00000000	6.70896×10^{-3}
5	40	0:00000000	1.38896×10^{-3}
6	48	0:00000000	2.20684×10^{-4}
7	56	0:00000000	2.72935×10^{-5}
8	64	0:00000000	2.80034×10^{-6}
9	72	0:00000000	2.85099×10^{-7}
10	80	0:01386749	3.74373×10^{-8}
11	88	0:03736518	6.55106×10^{-9}
12	96	0:04545455	1.292×10^{-9}

13	104	0:05200308	$2.61516 \times 10{-}10$
14	112	0:06394453	$5.31292 \times 10{-}11$
15	120	0:07010786	$1.07976 \times 10{-}11$
16	128	0:07241911	$2.1945 \times 10{-}12$
17	136	0:06741140	$4.46007 \times 10{-}13$
18	144	0:06086287	$9.06461 \times 10{-}14$
19	152	0:06163328	$1.84228 \times 10{-}14$
20	160	0:07665639	$\underline{3.74424 \times 10^{-15}}$

Table 4: Estimated values of Q (k) and P (K > k)

3.1.1 Estimating Probability of Lateral Overlap: Py (0)

P_y (0) is defined as the probability of lateral overlap of aircraft nominally flying at adjacent flight levels on same route. We can now use the same mixture model of Section 2.4 to compute this parameter by substituting $S_y = 0$ in the equation (2). This leads to an estimate of P_y (0) as 0.2.

However as noted earlier in the EUR/SAM report, this factor P_y (0) has a significant effect on the risk estimate. Therefore, it should not be underestimated. P_y (0) will increase as the lateral navigational performance of typical aircraft improves, causing a corresponding increase in the collision risk estimate. As reported in the EUR/SAM report, the RGCSP was aware of this problem and attempted to account for improvements in navigation systems when defining the RVSM global system performance specification. Based on the performance of highly accurate area navigation systems observed in European airspace, which demonstrated lateral path-keeping errors with a standard deviation of 0.3 NM, the RGCSP adopted a value of 0.059 as the value of P_y (0) for the global system performance.

As observed by many monitoring agencies and pointed out to us by AAMA, the RGCSP value of P_y (0) does not acknowledge the close track-keeping observed with RNP 4 or GNSS-equipped RNAV 10/RNP 10 aircraft. Further the EMA Handbook recommends to take a conservative value as 0.2. So we take this conservative value for our analysis as well.

3.1.2 Estimation of the parameter $\left|\overline{\dot{x}}\right|$

$\left|\dot{x}\right|$ is defined as the minimum relative along-track speed necessary for following aircraft in a pair separated by m NM at a reporting point to overtake lead aircraft at the next reporting point. Thus if d is the distance between the two way points and v_0 is the speed of the front aircraft then $\left|\dot{x}\right|$ can be computed by the equation.

$$\frac{d - m}{v_0} = \frac{d}{v_0 + \left|\overline{\dot{x}}\right|},$$

leading to

$$\left|\overline{\dot{x}}\right| = \frac{m v_0}{d - m}.$$

We conservatively estimate it by taking v_0 as the minimum speed observed in TSD which is 360 NM per hours and the maximum distance between two waypoints on the routes which we study which is d = 864 NM. With m = 50 NM the final estimate turns out to be $\left|\overline{\dot{x}}\right|$ = = 12.9496402877698 knots. We use a conservative value of 12 knots.

3.1.3 Estimation of the Parameter: $\overline{|\dot{y}(0)|}$

$\overline{|\dot{y}(0)|}$ is defined as the relative cross-track speed of same route aircraft pair. No data is available for estimation of this parameter so we take a conservative value of **1** knot as given in the EMA Handbook.

3.1.4 Estimation of the Parameter Q (k)

$Q(k)$ is defined as the proportion of aircraft pairs with initial longitudinal separation k. We estimate its value from the TSD. Flights entering the FIR on different routes and assigned different flight levels were considered separately (see Figure 5), and the waiting times between successive arrivals were tabulated in minutes. We assume an average speed of 8 NM per minute, and compute the proportion $Q(k)$ as

$$Q\ (k) = \frac{\text{number of flights pairs with inter-arrival distance 8k}}{\text{Total number of flight pairs with at least 50 NM separation}}$$
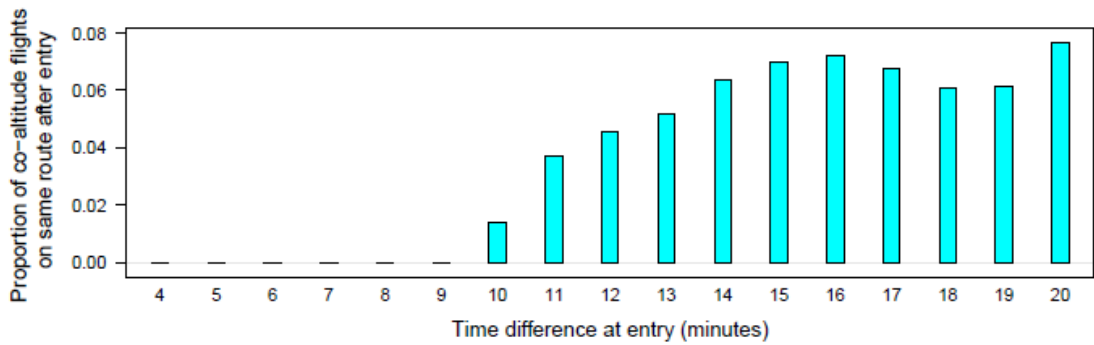

Figure 5: Values of $Q(k)$ estimated from TSD. For co-altitude flights on the same route (after entry / before exit), the proportion of flights that entered k minutes after the preceding flight is plotted for $k = 1, 2, 3, \ldots, 20$ minutes.

The final estimated values of $Q(k)$ for k ranging between 1 and 20 minutes is given in the Table 4.

3.1.5 Estimation of the Parameter $\mathbf{P}(K > k)$

To estimate $\mathbf{P}(K > k)$ we consider two aircrafts flying on same route at same flight levels at the same direction. Let V and V^t be their ground speeds of the front and behind aircraft respectively. We assume these speeds to be statistically independent but identically distributed. Let T_0 be the maximum duration of time before ATC intervenes. Then

$$P(K>k) = P(0< \frac{k}{V^t - V} <T_0) = P(V^t\text{-}V> \frac{k}{T_0})$$

We note here that the value of T_0 is conservatively taken to be 0.5 hours.

Now we finally estimate these probabilities using the TSD. For that we consider the difference in velocity of two aircraft nominally flying on the same route at the same flight level, after removing records with unusually high or low traversal times.

We observe that these differences in velocity are symmetrically distributed around zero but from the histogram and the quantile- quantile plot (see Figure 6) it is not clear that these differences necessarily

Normally distributed. To be conservative, we postulate the following mixture model for the density of these velocity differences.

$$f_v(v) = p\frac{\beta_v}{2}e^{-\beta_v|v|} + (1-p)\frac{1}{\sqrt{2\pi}\sigma_v}e^{-\frac{v^2}{2\sigma_v^2}},$$

which is a mixture of Double Exponential and Normal densities with mixing proportion *p*.

We then estimate the parameters of this mixture model by their *maximum likelihood estimates (MLEs)*. Since this is a mixture model so we use the *Expectation-Maximization (EM)* algorithm to find the MLEs. The algorithm converged rapidly to give the following estimates:

$$\widehat{p} = 0.5179022 \qquad \widehat{\beta}_v = 0.09958575 \qquad \widehat{\sigma}_v = 28.74828$$
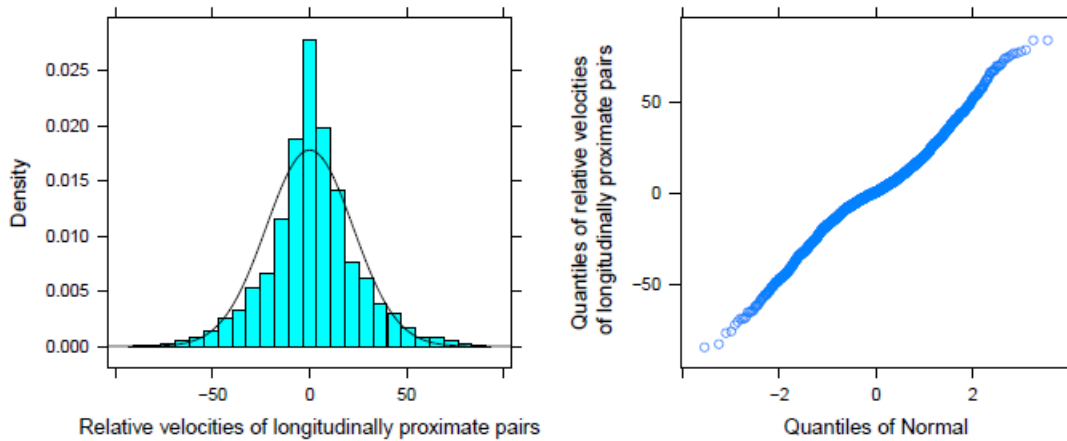

Figure 6: Distribution of relative velocities of longitudinally proximate pairs. The Normal distribution does not necessarily seem to be a reasonable fit.

It is well known in Statistics literature that even though the EM algorithm increases the value of the likelihood it may get trapped in a local maximum. To avoid this problem we tried several starting values and observed that the algorithm always converges to the same estimated values given above.

So it is statistically reasonable to accept the mixing density with this value of the parameters as a good estimate of the true density of the velocity differences. A graphical representation of the fit is given in Figure 7.

With these we estimate the values of $\mathbf{P}(K > k)$ for *k* ranging between 7 and 20. These are presented in the Table 4.
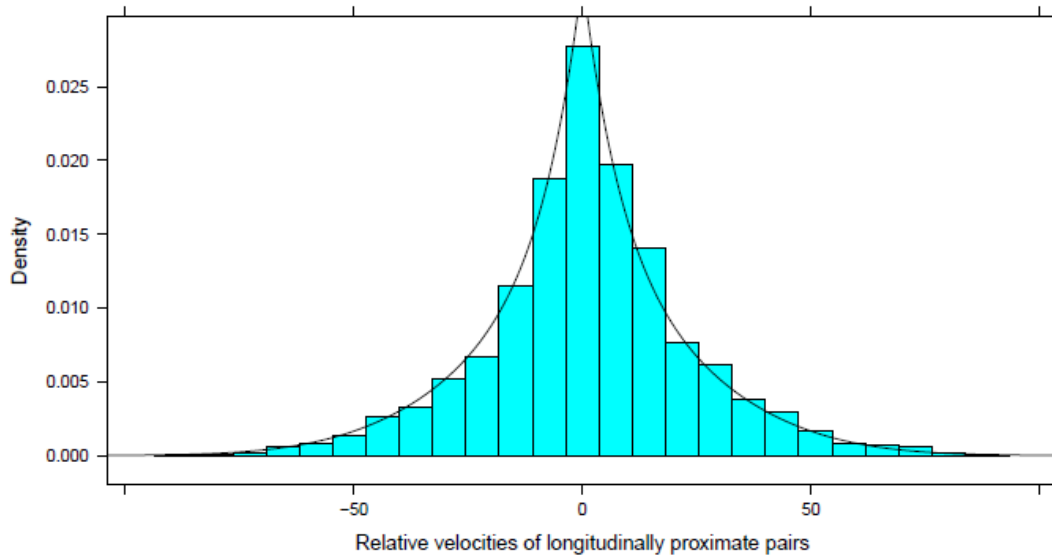
Figure 7: Distribution of relative velocities of laterally proximate pairs along with estimated mixture density (estimated using the EM algorithm).

3.2 Estimated Values of the Parameters and Estimated Longitudinal Collision Risk for Routes with 50 NM Longitudinal Separation

The following table gives the values of the parameters of the right-hand side of the equation (4) which are obtained from our analysis.

Parameter	Estimated Values	Source of the Estimate
m	50 NM	Current minimum longitudinal separation (due to RHS).
M	160 NM	Conservative value corresponding to 20 minutes separation.
λx	0.0338717 NM	Estimated using flights on 50NM routes only.
λy	0.03176976 NM	Estimated using flights on 50NM routes only.
λz	0.009423522 NM	Estimated using flights on 50NM routes only.
$P_y(0)$	0.2	Conservative estimate (see Section 3.1.1).
$P_z(0)$	0.538	Conservative value used in previous safety assessments (see Section 2.5).
$|\dot{x}|$	24 knots	Conservative estimate using speed and distance between way points (see Section 3.2.1).
$|\dot{y}(0)|$	1 knot	RASMAG/9 safety assessment (see Section 3.1.3).
$|\dot{z}|$	1.5	Conservative value as per EMA Handbook (see Section 2.9).
$Q(k)$	See Table 5	Obtained from TSD (see Section 3.2.2).
$\mathbf{P}(K > k)$	See Table 5	Computed using normal model on speed (see Section 3.1.5).

21

Finally this leads to the following estimate for the longitudinal collision risk N_{ax}^{50}

$$N_{ax}^{50} = 3.97378 \times 10^{-9}$$

3.2.1 Estimation of the parameter $\left|\dot{\bar{x}}\right|$

Estimation of $\left|\dot{\bar{x}}\right|$ is performed in the same manner as described in Section 3.2.1, with m = 50. The final estimate turns out to be $\left|\dot{\bar{x}}\right|$ = 24.4233378561737 knots. We use a conservative value of 24 knots.

3.2.2 Estimation of the Parameter $Q(k)$

$Q(k)$ is defined as the proportion of aircraft pairs with initial longitudinal separation k. We estimate its value from the TSD, using only flights on routes with 50 minutes separation (see Figure 5).

The final estimated values of $Q(k)$ for k ranging between 1 and 20 minutes is given in the Table 5.

k	k (NM)	$Q(k)$	$P(K>k)$
1	8	0:000000000	2.20864×10^{-1}
2	16	0:000000000	8.87478×10^{-2}
3	24	0:000000000	2.9282×10^{-2}
4	32	0:000000000	7.57888×10^{-3}
5	40	0:000000000	1.52418×10^{-3}
6	48	0:000000000	2.45511×10^{-4}
7	56	0:000000000	3.52971×10^{-5}
8	64	0:000873744	5.61588×10^{-6}
9	72	0:002184360	1.15036×10^{-6}
10	80	0:058540848	2.79285×10^{-7}
11	88	0:050240280	7.14776×10^{-8}
12	96	0:068152031	1.84729×10^{-8}
13	104	0:073394495	4.78006×10^{-9}
14	112	0:077326343	1.23703×10^{-9}
15	120	0:079947575	3.20132×10^{-10}
16	128	0:090432503	8.28472×10^{-11}
17	136	0:069899519	2.14401×10^{-11}
18	144	0:075141983	5.54851×10^{-12}
19	152	0:065967671	1.4359×10^{-12}
20	160	0:074705111	$\underline{3.71599 \times 10^{-13}}$

Table 5: Estimated values of $Q(k)$ and $\mathbf{P}(K > k)$

3.2.3 Estimation of the Parameter $\mathbf{P}(K > k)$

As in Section 3.1.5 we estimate these probabilities using the TSD by considering the difference in velocity of two aircraft nominally flying on the same route at the same flight level, after removing records with unusually high or low traversal times.

We observe that these differences in velocity are symmetrically distributed around zero but from the histogram and the quantile- quantile plot (see Figure 9) it is not clear that these differences necessarily   Normally distributed. To be conservative, we postulate the following mixture model for the density of these velocity differences.

$$f_v(v) = p\frac{\beta_v}{2}e^{-\beta_v|v|} + (1-p)\frac{1}{\sqrt{2\pi}\sigma_v}e^{-\frac{v^2}{2\sigma_v^2}},$$

which is a mixture of Double Exponential and Normal densities with mixing proportion *p*.

We then estimate the parameters of this mixture model by their *maximum likelihood estimates (MLEs)*. Since this is a mixture model so we use the *Expectation-Maximization (EM)* algorithm to find the MLEs. The algorithm converged rapidly to give the following estimates:

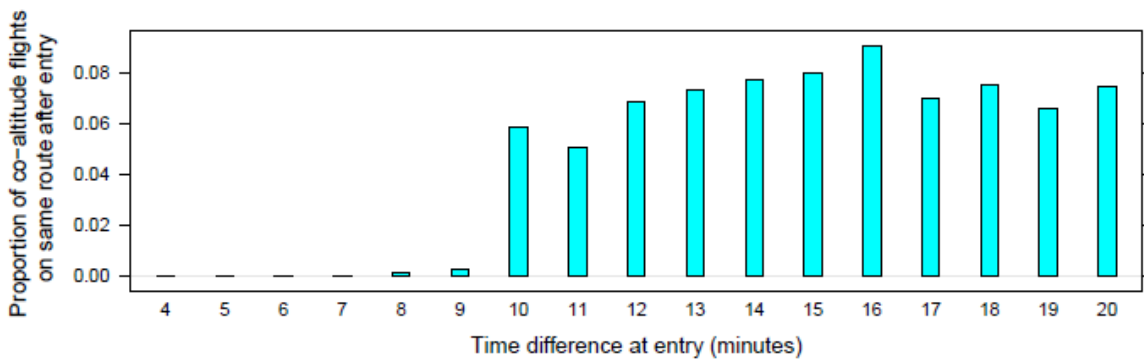$$\hat{p} = 0.4093749 \qquad \hat{\beta}_v = 0.08448341 \qquad \hat{\sigma}_v = 28.05924$$

Figure 8: Values of Q(k) estimated from TSD. For co-altitude flights on the same route   (after entry / before exit), the proportion of flights that entered k minutes after the preceding flight is plotted for k = 1, 2, 3, . . . , 20 minutes.

A graphical representation of the fit is given in Figure 10.

With these we estimate the values of $\mathbf{P}(K > k)$ for *k* ranging between 7 and 20. These are presented in the Table 5.

4.   Summary of the Safety Assessment

The following table gives a summary of the safety assessment of the BOBASIO region for the month of December 2014.

Type of Risk	Estimated Risk	TLS	Remarks
Lateral Risk	1.69506×10^{-9}	$\mathbf{5 \times 10^{-9}}$	Below TLS
Longitudinal Risk (30 NM Routes)	0.138395×10^{-9}	$\mathbf{5 \times 10^{-9}}$	Below TLS
Longitudinal Risk (50 NM Routes)	3.97378×10^{-9}	$\mathbf{5 \times 10^{-9}}$	Below TLS

Figure 11 presents the results of the collision risk estimates for each month using the cumulative 12-month LLD reports since January 2015.
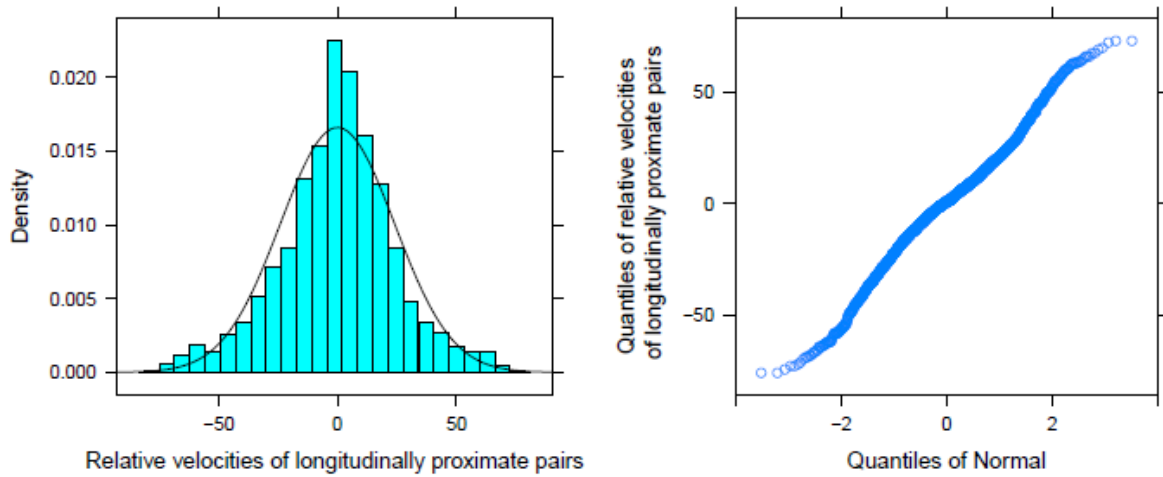
Figure 9: Distribution of relative velocities of longitudinally proximate pairs. The Normal distribution does   not necessarily seem to be a reasonable fit.
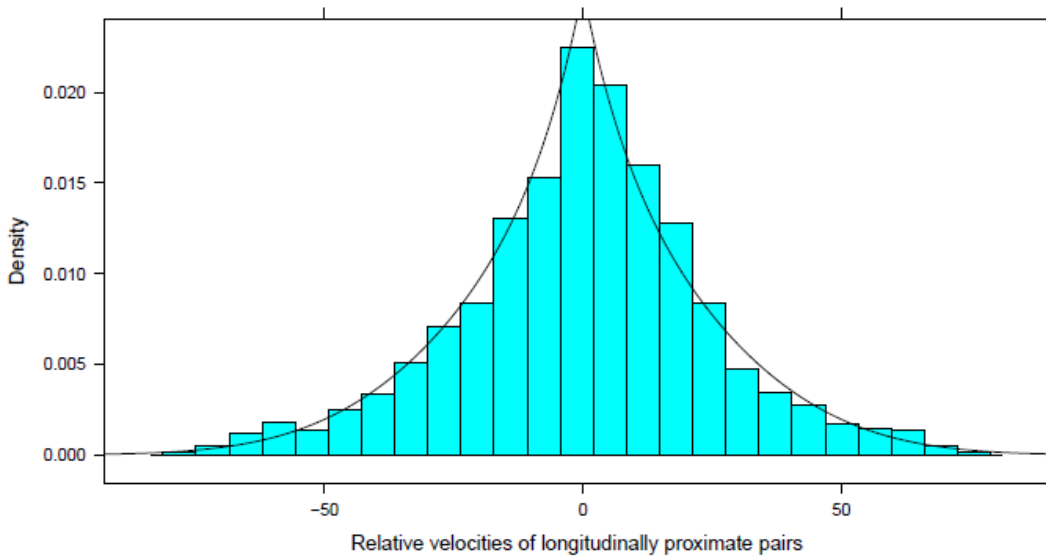

Figure 10: Distribution of relative velocities of laterally proximate pairs along with estimated mixture density   (estimated using the EM algorithm).
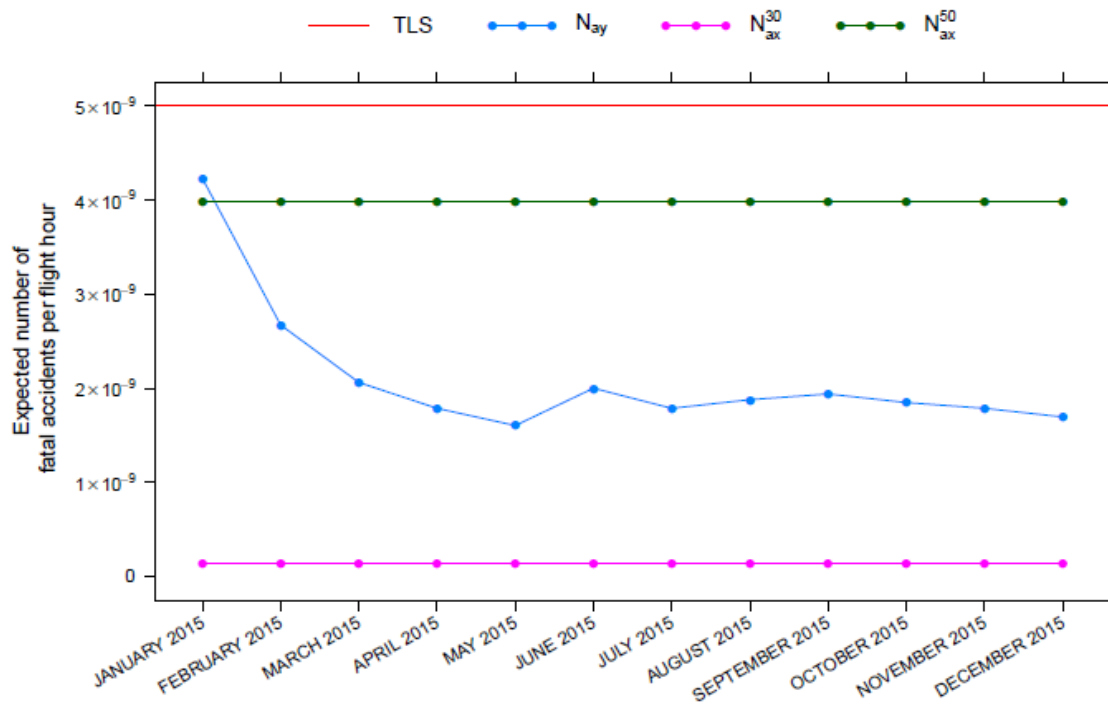
Figure 11: Assessment of Compliance with Lateral and Longitudinal TLS Values.